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AIRPLANE STABILITY CALCULATIONS

and Their Verification by Flight Tests.

By Augusto Rota
Italian Naval Engineer.

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Questions relating to the stability of airplanes in flight have greatly interested theorists since the beginning of aviation and there is no one who is not acquainted with the remarkable works of Mr. Painleve, Prof. Bryan, Col. Crocco and many others.

Unfortunately nearly all these works were only veryremarkable solutions of a very interesting theoretical mechanical problem, but of very little importance from the practical point of view.

In fact, constructors proportion their airplanes, in their organs of equilibrium and stabilization, according to practical data acquired by years of observation and successive improvements of old airplanes, with little thought of the results of the discussion of the systems of differential equations established by scientists.

For some time, however, they have begun to occupy themselves with questions connected with longitudinal stability. As was quite natural, they have sought to divest the theories of everything complicated, in order to employ the simplest and also the surest data. Thus, in a manner similar to that followed by the engineers of naval construction, they have attached the greatest importance to the coefficient of initial longitudinal stability, i.e., to the quantity

$$C_s = \frac{\delta M}{\delta \alpha}$$

*From "Premier Congres International de la Navigation Aerienne," Paris, November, 1921, Vol. I, pp. 42-48.

in which they consider

$$M = M\ (V^2\delta,\ \alpha,\ \tau,\ \varphi).$$

M being the moment with respect to the center of gravity of the airplane at the speed V, the relative density δ the angle of attack α the angle of deviation of the elevators τ and the angle of flight φ.

It was very natural to consider also the quantity

$$H = \frac{1}{\pi}\ \frac{\delta M}{\delta \alpha}$$

π being the weight of the airplane, H the "virtual metacentric height" and

$$h = \frac{H}{V^2\delta}$$

h receiving the designation of "reduced metacentric height."

The terms C^s, H and h must have positive values, so that the airplane will be stable in its longitudinal motion. The consideration of the term H is justified by analogy with the corresponding term in the theory of the ship, that of the term h in the fact that the action of all the organs of control and equilibrium of the airplane is proportional to the term $V^2\delta$.

To all these quantities however there has hitherto been generally given only a qualitative importance,* not only because it was not yet very well known what value to give them in comparison with the other characteristics of an airplane, but also because

* We know, for example, the diagrams employed by Mr. Eiffel and the displacements of the center of gravity for different values of the angle τ. These diagrams answer quite well for the qualitative solution of the problem of stability.

the deduction of the results of experiments on models of the values of

$$\frac{\delta M}{\delta \alpha}$$

is not made very easily, especially when the position of the center of gravity has not yet been fixed.

It was for the purpose of eliminating this second disadvantage that we thought of utilizing, for the quantitative study of the stability of airplanes, a diagram which is very simple to construct from the data of the tunnel tests and which neither depends on the position of the center of gravity nor of the angle of deflection of the elevators. This diagram is constructed by means of straight lines drawn through the metacenters of the complete airplane, in a direction parallel to the tangents to the polar of the airplane relative to a system of axes fixed with reference to the airplane (Fig. 1).

If m is the metacentric curve of the airplane corresponding to a given fixed position of the elevator, if p is the polar with reference to a system of axes fixed with respect to the airplane, and if M is the metacenter corresponding to an angle of attack α, to which the slope of flight φ, the total action of the air on the airplane F is on the straight line tangent at M to the metacentric curve, parallel to the corresponding vector OK of the polar.

If the angle α, without changing the speed nor the direction of flight, becomes $\alpha' = \alpha + \Delta\alpha$, the action of the air on the airplane becomes F', the resultant of F and ΔF,

$\Delta F'$ being, if $\Delta\alpha$ is very small, on the straight line d drawn through M parallel to the tangent at K to the polar p.

From this it is deduced that the moment ΔM, which is produced by the rotation $\Delta\alpha$ of the airplane, is equal to $\Delta F \times c$, c being the distance of the C.G. from the line d.

We then have, at the limit for $\Delta\alpha = 0$:

$$C_s = \frac{\Delta F}{\Delta\alpha} \times c$$

and

$$H = \frac{1}{\pi} \frac{\Delta F}{\Delta\alpha} c$$

or, by approximation

$$H = \frac{1}{F} \frac{\Delta F}{\Delta\alpha} \times c$$

and, if the resultant of K_n and K_y, or of K_t and K_n, is indicated by K:

$$H = \frac{1}{K} \frac{\Delta K}{\Delta\alpha} \times c$$

lastly

$$h = \frac{1}{KV^2\delta} \frac{\Delta K}{\Delta\alpha} \times c$$

or

$$h = \frac{\frac{\Delta K}{\Delta\alpha}}{\frac{\pi}{S}} \times c$$

from which it follows that h is proportional to c, $\frac{\Delta K}{\Delta\alpha}$ being nearly constant for all angles practically accessible in flight and $\frac{\pi}{S}$ being constant for each airplane. After this is established, it is possible to substitute, in the study of stability, the consideration of c for that of h, c having on its part a more evident geometrical and mechanical signification. To this c we have given the name "index stability."

We will see immediately that the position of the line d, which depends on the angle of attack and the slope of flight, does not depend on the angle of deviation of the elevators. In fact, a variation $\Delta\tau$, of the angle of deviation τ at constant speed, produces a variation of only ΔF_p of the force of the air on the horizantal tail planes, which remains the same with reference to the airplane, when the latter is tilted by the small angle $\Delta\alpha$ already considered. Consequently, both forces F and F' are displaced by combining with the same force ΔF_p, but their geometrical difference does not vary.

Consequently, not only the lines d can all be determined by a single position of the elevators, no matter whether the latter is compatible or not with the equilibrium at the different angles of attack, but they remain unchanged for every slope of flight and angle of attack, even if the position of the C.G. or the axis of traction is changed. In fact, the displacement of the C.G. or the axis of traction has no other effect on the whole airfoil of the airplane than a variation of the angle of deviation of the elevators, which has no influence on the lines d.

The lines d (each of which corresponds to a given value of α and of the slight slope φ and the distance of which from the C.G. is the index of stability) are therefore very useful in the study of the stability of an airplane, even before knowing the position of the C.G., and can be utilized in this case for determining the most suitable location for this point.

The construction of all the lines d^* is very simple in accordance with the results of the tunnel tests of the complete airplane or of just the cell and tail. In fact, the direction of each straight line is that of the tangent to the relative polar through the point which corresponds to the angle of attack under consideration. The distance of the intersection of the line d with the axis of the airplane from the point of this axis, with reference to which the moments are measured, is given by the expression

$$c' = \frac{d\,M}{dK_n}\,l$$

in which M is the coefficient of moment with reference to the chosen point, K_n is the component following the normal to the axis of the coefficient of total thrust K; and l is the wing chord.

Now, M is composed of a part M_c, due to the action of the air on the cell, and a part M_p, due to the tail planes. We may write

$$M_p = xK_{np}\,\frac{S_p}{S}\,t$$

in which K_{np} is the coefficient of normal thrust relative to the horizontal tail planes (function of τ and α); S_p the surface of the tail planes; t the distance (which may be considered constant) of the thrust K_{np} from the reference point of the mo-

*We have given to the lines d the name of "metacentric lines." In fact, each one is the locus of the metacenters of the airplane corresponding to each slope and angle of attack for different values of τ.

ments. x is the ratio of the square of the velocity of the air with relation to the tail planes at the speed V of the airplane. x only needs to be considered when the tail planes are in the slip stream from the propeller and it is a determined function of $\frac{V}{ND}$ or perhaps of $\frac{P}{V^2 \delta}$, p being the propeller thrust, or again of $Kx + Ky \tan \varphi$, i.e., an increasing function of the angle of attack and the slope of flight. We may write, according to Froude's theorem,

$$x = 1 + \frac{S}{S'_e} \frac{K_x + K_y \tan \varphi}{\frac{a_o}{2}}$$

in which S'_e is a suitable fraction of the surface of the propeller and a_o the density of the air. We have

$$c' = \frac{dM_c}{dK_n} l + \frac{x \epsilon \frac{\delta K_{np}}{\delta \alpha_p}}{\frac{dK_n}{d\alpha}} \frac{S_p}{S} t$$

in which $\frac{\delta K_{np}}{\delta \alpha_p}$ is dependent on the shape of the tail planes, a constant which is equal to the complement to the unity of the ratio, nearly constant, of the angle of deviation of the air current of the cell to the angle of attack α of the cell, measured from the angle corresponding to the zero thrust.

The result is that the sum of a term $c'_c = \frac{d M_c}{d K_n}$ which depends only on the cell and of a term

$$c'_p = \frac{n \epsilon \frac{\delta K_{np}}{\delta \alpha_p}}{\frac{d K_n}{d \alpha}} \frac{S_p}{S} t$$

which, by x constant, is constant at all angles of attack and depends on the relation of certain characteristics of the tail planes to the corresponding characteristics of the cell, so that all the lines d relating to the airplane are obtained on displacing, by a determined amount c_p (constant, if x is constant), all the lines d relative to the single cell.

Nothing is therefore more simple than the construction of the lines d, especially if we are not limited first to the consideration of the values of x equal to 1, i.e., if we do not neglect the increase in efficacy of the tail planes due to the slip stream, which leaves us to consider first only the gliding stability.

When we have the diagram of the lines d, nothing is simpler than the study of the longitudinal stability of the airplane. It is only necessary to measure the distance of the C.G. from the lines d corresponding to the conditions of flight, in order to obtain the values of the "index of stability."

If we cannot place the C.G. in a suitable position and if we are obliged to displace the lines d, the established formulas give very simply the variations of the characteristics of the tail planes necessary for obtaining this displacement.

Figs. 2 and 3 show the lines d for x = 1 for a separate wing (Fig. 2) and for a complete airplane (Fig. 3).

This being established, there also results a notable simplification in the interpretation of the tests which we proposed in 1919 for the verification, in full flight, of the calculations of stability, if we apply the latter to the determination of the quantities c.

In fact, if we consider two speeds very near stable flight determined by the angles of attack α and $\alpha + \Delta\alpha$ and by the slopes φ and $\varphi + \Delta\varphi$, we have in the second speed the following variations of the forces (with reference to the unity of speed and the surface of the airplanes): the force ΔK acting along the line d, the force $\Delta \frac{P}{SV^2\delta} = \Delta (K_x + K_y \tan\varphi)$ acting on the axis of the propeller, at the distance e from the C.G., the force $\frac{S_p}{S} K_{np} \Delta x$ acting on the tail planes.

If, as in the general case, these forces do not offset each other, it is necessary in the second case to deflect the elevators by an angle $\Delta\tau$, so as to produce a force $\frac{\delta K_{np}}{\delta\tau} \frac{S_p}{S} \alpha\tau$ whose moment will offset that of the forces considered above, i.e., it is necessary that

$$- t x \frac{\delta K_{np}}{\delta\tau} \frac{S_p}{S} \Delta\tau = d \Delta K + e \Delta (K_x + K_y \tan\varphi) + t \frac{S_p}{S} K_{np} \Delta x$$

whence, at the limit, and on putting $\Delta K = \Delta K_y$

$$c = - x \frac{\delta K_{np}}{\delta\tau} \frac{S_p}{S} t \frac{d\tau}{dK_y} - e \frac{d(K_x + K_y \tan\varphi)}{dK_y} - K_{np} \frac{S_p}{S} \frac{dx}{dK_y}$$

If, therefore, we observe in flights at certain speeds, the values of τ and if we determine the corresponding values of $K_x + K_y \tan\varphi$, K_{np} and x, by choosing a series of flights which may be considered as continuous, e.g., the series corresponding to constant values of φ (Fig. 4), we can calculate, by very simple processes of graphic derivation, the values of c corresponding to each flight speed.

If we choose the series for which α is constant, we have $\Delta K = 0$ and

$$\Delta (K_x + K_y \tan \varphi) = K_y \Delta\varphi$$

$$- t x \frac{\delta K_{np}}{\delta \tau} \frac{S_p}{S} \Delta \tau = e K_y \Delta\varphi + t \frac{S_p}{S} K_{np} \Delta x$$

whence, at the limit,

$$- t \frac{S_p}{S} K_{np} = \frac{d\varphi}{dx} \left(tx \frac{\delta K_{np}}{\delta\tau} \frac{S_p}{S} \frac{d\tau}{d\varphi} - e K_y \right)$$

which renders it possible by analogous methods to evaluate the term $t \frac{S_p}{S} K_{np}$ which would be more difficult by direct methods.

If, according to Froude's theorem, which is admissible, we put

$$x = 1 + \frac{S}{S'_e} \frac{K_x + K_y \tan\varphi}{\frac{a_o}{2}}$$

we have

$$\Delta x = \frac{S}{S'_e} \frac{\Delta(K_x + K_y \tan \varphi)}{\frac{a_o}{2}}$$

which enables the following very considerable simplification of the formulas established above

$$c = - x \frac{\delta K_{np}}{\delta \tau} \frac{S_p}{S} t \frac{d\tau}{dK_y} - \left(e^{e} + \frac{K_{np}}{\frac{a_o}{2}} \frac{S_p}{S'_e} t \right) \frac{d(K_x + K_y \tan \varphi)}{dK}$$

and for α constant

$$e + \frac{K_{np}}{\frac{a_o}{2}} \frac{S_p}{S'_e} t = - t x \frac{\delta K_{np}}{\frac{\delta \tau}{K_y}} \frac{S_p}{S} \frac{d\tau}{\delta \varphi}$$

After many and long attempts, which have taken considerable time and energy for perfecting the instruments and the general plan of the experiments, we have begun in Italy the study and systematic determination of the characteristics of longitudinal stability of airplanes in full flight, according to the method outlined above.

It is very probable that from these experiments, which will be tried also with airplanes known not to be very satisfactory in piloting, we will be able to draw quite clear conclusions concerning the values which are, for the different types, the most favorable for the good behavior of airplanes in flight and on the comparison of values obtained by tunnel tests.

Calculations of longitudinal stability of airplanes can then be made with confidence and profit, because we shall have eliminated and clarified everything that has hitherto been complicated, obscure and indeterminate.

Translated by the National Advisory Committee for Aeronautics.

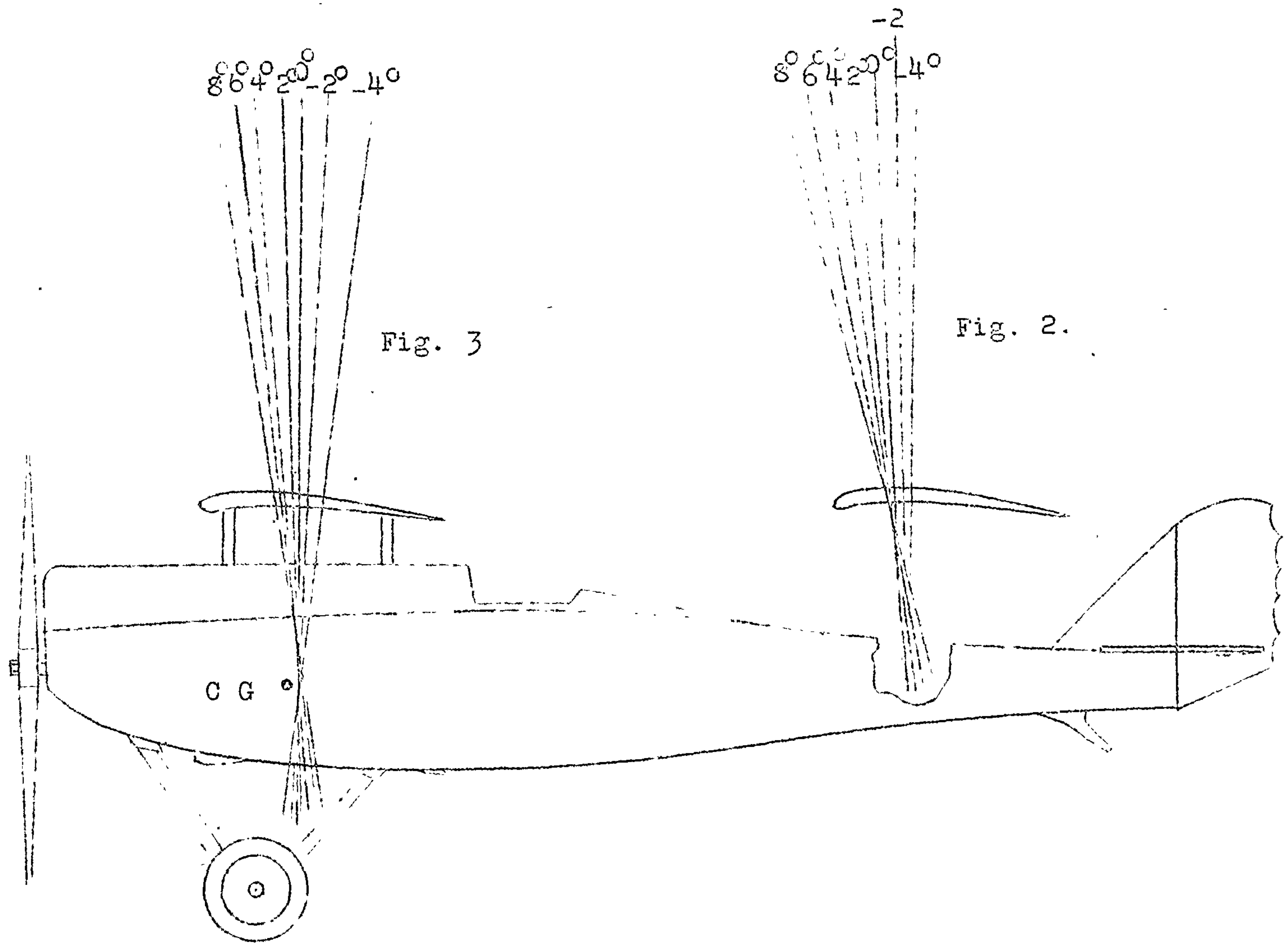

Fig. 3

Fig. 2.

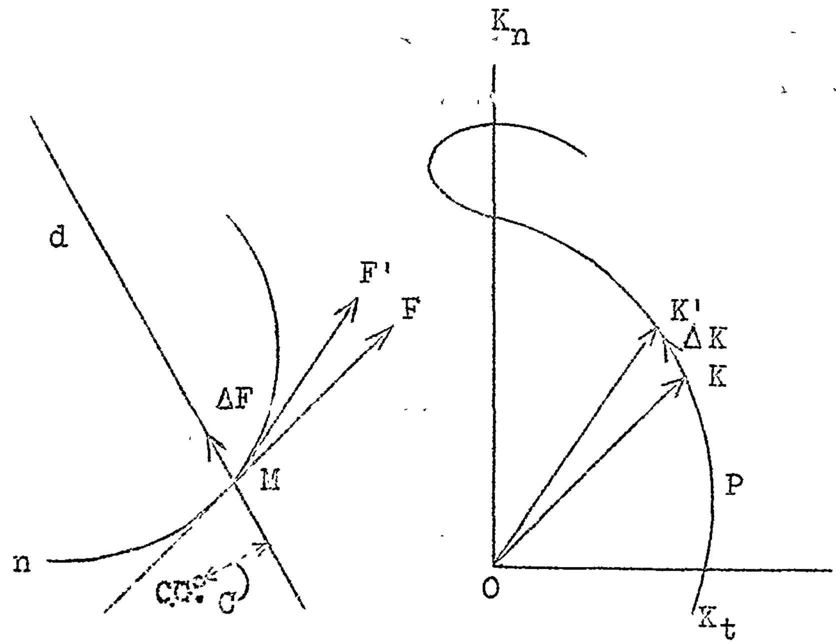

Fig. 1.

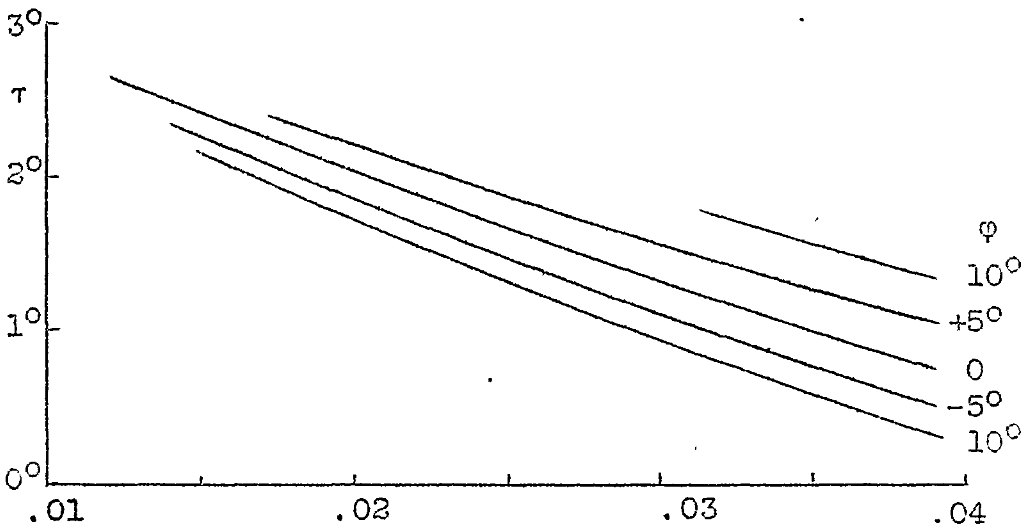

Fig. 4